56-3-86



# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3370

## A SIMPLIFIED METHOD FOR CALCULATING AEROELASTIC EFFECTS ON THE ROLL OF AIRCRAFT

By John M. Hedgepeth, Paul G. Waner, Jr., and Robert J. Kell

Langley Aeronautical Laboratory
Langley Field, Va.

NACA

Washington
March 1955

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3370

# A SIMPLIFIED METHOD FOR CALCULATING AEROELASTIC EFFECTS ON THE ROLL OF AIRCRAFT

By John M. Hedgepeth, Paul G. Waner, Jr., and Robert J. Kell

## SUMMARY

An approximate linearized lifting-surface theory is used in conjunction with structural influence coefficients to formulate a method for analyzing the aeroelastic behavior in roll of an aircraft. Rolling effectiveness and aileron-reversal speed are computed by the use of a Galerkin-type procedure. Results obtained for two example configurations by using this method are compared with the results obtained by using the more refined method of NACA TN 3067. The agreement is excellent.

## INTRODUCTION

In the design of modern high-speed aircraft, it is generally recognized that aeroelastic effects must be accounted for accurately. One method which should be capable of yielding reliable predictions of the aeroelastic effects on the roll of supersonic aircraft has been presented in reference 1. This method, which makes use of structural influence coefficients to determine the distortions and lifting-surface theory to determine the airloads, involves, however, a considerable amount of computational labor. For this reason, some means for simplifying the computations without introducing an objectionable amount of error was sought. The purpose of this paper is to describe the resulting simplified method and to evaluate its accuracy.

In this paper, attention is confined to the rolling problem. The actual aircraft configuration is left general, the only restriction being that the effects of chordwise bending are assumed to be negligible. Both subsonic and supersonic speeds can be treated by the method, but particular attention is paid to the supersonic regime in the examples.

## SYMBOLS

| | |
|---|---|
| $A$ | parameter defined by equation (13a) |
| $B_\theta, B_p, B_\delta$ | parameters defined by equations (13b) |
| $C_{l_\theta}, C_{l_p}, C_{l_\delta}$ | rolling-moment derivatives defined by equations (13c) |
| $GJ$ | elementary torsional stiffness of wing |
| $G_L(y,\eta)$ | structural-twist influence function due to unit concentrated load at y-axis |
| $G_M(y,\eta)$ | structural-twist influence function due to unit concentrated torque |
| $L(y)$ | aerodynamic load per unit span, positive upward |
| $M(y)$ | aerodynamic pitching moment about y-axis per unit span, positive in positive twist direction |
| $M_o$ | free-stream Mach number |
| $P_h$ | static pressure at altitude |
| $P_O$ | standard static pressure at sea level |
| $Q(y)$ | aerodynamic pitching moment about elastic axis per unit span, positive in positive twist direction |
| $V$ | free-stream velocity |
| $a$ | ratio of fuselage radius to exposed semispan of wing |
| $b$ | total wing span, $2(al + l)$ |
| $c(y)$ | wing chord |
| $\bar{c}$ | mean geometric chord |
| $c_a$ | aileron chord |
| $c_l$ | section lift coefficient, $L(y)/qc$ |

| | |
|---|---|
| $c_m$ | section pitching-moment coefficient about y-axis, $M(y)/qc^2$ |
| $c_q$ | section pitching-moment coefficient about elastic axis, $Q(y)/qc^2$ |
| $e(y)$ | distance measured forward from y-axis to elastic axis, expressed as fraction of local chord |
| $k$ | amplitude of twist mode shape |
| $l$ | exposed wing semispan |
| $p$ | rolling angular velocity, positive in right-hand sense |
| $pb/2V$ | tangent of wing-tip helix angle |
| $q$ | dynamic pressure |
| $x,y$ | coordinate system (see fig. 1) |
| $\beta$ | cotangent of Mach angle, $\sqrt{M_o^2 - 1}$ |
| $\theta(y)$ | angle of twist of wing (see fig. 1) |
| $\theta_1(y)$ | twist mode shape used in Galerkin-type procedure |
| $\phi$ | rolling effectiveness, $(pb/2V)_F/(pb/2V)_R$ |
| $\delta$ | aileron deflection (see fig. 1) |
| $\gamma$ | ratio of specific heats |

Subscripts:

| | |
|---|---|
| F | flexible wing |
| R | rigid wing |
| $\alpha_\theta$ | effective aerodynamic coefficients due to twist |
| p | parameters or aerodynamic coefficients due to unit $pb/2V$ |
| rev | aileron reversal |
| $\delta$ | parameters or aerodynamic coefficients due to unit aileron deflection |

$\theta$ parameters or aerodynamic coefficients due to unit twist shape

Matrix notation:

$[\ ]$ square matrix

$\lfloor\ \rfloor$ row matrix

$|\ |$ column matrix

$[\diagdown]$ diagonal matrix

## ANALYSIS

The analysis proceeds along the same lines as that in reference 1; that is, the structural deformations are expressed in terms of the airloads, the airloads are obtained, and then the two are combined to formulate the aeroelastic problem.

### Structural Deformations

Consider the configuration shown in figure 1. If the effects of chordwise bending are assumed to be negligible, the only distortion of interest in this problem is the twist $\theta(y)$ which can be expressed in terms of the section lift $L(y)$ and the section moment about the y-axis $M(y)$ as follows:

$$\theta(y) = \int_0^l G_L(y,\eta)\ L(\eta)\ d\eta + \int_0^l G_M(y,\eta)\ M(\eta)\ d\eta \tag{1}$$

where $G_L(y,\eta)$ and $G_M(y,\eta)$ are influence functions which define the wing twist at $y$ due to a unit concentrated load at the y-axis and a unit concentrated torque, respectively, at the spanwise station $\eta$. As was pointed out in reference 1, these influence functions can be found either theoretically (refs. 2, 3, and 4) or, if necessary, experimentally.

If an elastic axis (defined as a line along which loads can be placed without producing significant twist anywhere) exists, this equation can be simplified to be

$$\theta(y) = \int_0^l G_M(y,\eta)\ Q(\eta)\ d\eta \tag{2}$$

The quantity $Q(y)$ is the section torque about the elastic axis and is given by

$$Q(y) = M(y) - e(y)\ c(y)\ L(y) \tag{3}$$

in which $e(y)$ is the distance measured forward from the y-axis to the elastic axis, expressed as a fraction of the local chord.

## Aerodynamic Loads

The section lift and pitching moment can be expressed in coefficient form as

$$\left.\begin{aligned} L(y) &= q\ c(y)\ c_l(y) \\ M(y) &= q\ c^2(y)\ c_m(y) \end{aligned}\right\} \tag{4}$$

By assuming linearity of the aerodynamics, the loading coefficients for steady roll can be written as

$$\left.\begin{aligned} c_l(y) &= c_{l_{\alpha_e}}(y)\ \theta(y) + c_{l_p}(y)\ \frac{pb}{2V} + c_{l_\delta}(y)\ \delta \\ c_m(y) &= c_{m_{\alpha_e}}(y)\ \theta(y) + c_{m_p}(y)\ \frac{pb}{2V} + c_{m_\delta}(y)\ \delta \end{aligned}\right\} \tag{5}$$

The principal way in which this analysis departs from that of reference 1 is in the manner of obtaining the loads due to structural deformation (the first term on the right-hand side of eqs. (5)). In the method of reference 1, the loads due to the arbitrary angle-of-attack distributions which arise from structural distortion were determined by an exact application of lifting-surface theory; in the present method, these loads are calculated approximately as can be seen from the following development. Consider, for example, the meaning of $c_{l_{\alpha_e}}(y)$. This function is actually the ratio of the section lift coefficient due to structural distortion to the angle of twist $\theta(y)$. In general, $c_{l_{\alpha_e}}(y)$ is dependent, in an apparently complex fashion, on the shape of the twist curve; different shapes yield different values of this function. Fortunately, however, the value of $c_{l_{\alpha_e}}(y)$ is relatively insensitive to changes in the shape of $\theta(y)$. This fact suggests that $c_{l_{\alpha_e}}(y)$ can be adequately approximated by calculating it for an angle-of-attack distribution that is reasonably close to the expected actual mode shape and then by considering this quantity to be fixed with respect to changes in the mode shape. This procedure of using an effective lift-curve slope, which has been used in the past by many investigators (for example, see ref. 5), obviously allows a considerable simplification of aeroelastic analyses.

For the rolling problem, the section rolling derivatives $c_{l_p}(y)$ and $c_{m_p}(y)$, which must be determined for use in equation (5), can also serve as a convenient basis for determining $c_{l_{\alpha_e}}(y)$ and $c_{m_{\alpha_e}}(y)$. The angle-of-attack distribution that yields these coefficients,

$$\alpha(y) = -\frac{al + y}{(1 + a)l} \tag{6}$$

is, for the present aerodynamic purposes, a fair approximation to the actual expected mode shape provided that a, the ratio of the fuselage radius to the exposed semispan of the wing, is not too large. Thus, the following expressions for $c_{l_{\alpha_e}}$ and $c_{m_{\alpha_e}}$ are used:

$$\left.\begin{aligned} c_{l_{\alpha_e}}(y) &= -\frac{(1 + a)l}{al + y} c_{l_p}(y) \\ c_{m_{\alpha_e}}(y) &= -\frac{(1 + a)l}{al + y} c_{m_p}(y) \end{aligned}\right\} \tag{7}$$

It follows from the foregoing development that the only aerodynamic information necessary for the aeroelastic analysis of the rolling problem is the section rolling and aileron derivatives. At supersonic speeds, these quantities are readily obtainable for most reasonable configurations. For rectangular wings, a rather complete derivation of the section lift and pitching-moment coefficients due to rolling and aileron deflection is included in reference 1. Lift and pitching-moment distributions due to roll can be obtained from references 6 and 7 for a wide variety of plan forms; the lift distributions are given directly, and the pitching-moment distributions can be found by proper integration of pressure distributions. Aileron loads can be found by methods such as those illustrated in reference 8; in some cases, two-dimensional strip theory should be adequate.

For subsonic speeds, no such complete coverage has been made. In the first place, all the theoretical approaches are approximate to some extent. In addition, not nearly so large a variety of plan-form shapes has been analyzed. However, papers such as reference 9 afford a considerable amount of help in finding the desired aerodynamic derivatives.

## Aeroelastic Equations

If the expressions for the loads (eqs. (4) and (5)) are substituted into the equilibrium equation (eq. (1)), the following aeroelastic equation results:

$$\theta(y) = q\int_0^l \left[c\ G_L(y,\eta)\ c_{l_{\alpha_e}}(\eta) + c^2\ G_M(y,\eta)\ c_{m_{\alpha_e}}(\eta)\right]\theta(\eta)\ d\eta\ +$$

$$q\ \frac{pb}{2V}\int_0^l \left[c\ G_L(y,\eta)\ c_{l_p}(\eta) + c^2\ G_M(y,\eta)\ c_{m_p}(\eta)\right]d\eta\ +$$

$$q\delta\int_0^l \left[c\ G_L(y,\eta)\ c_{l_\delta}(\eta) + c^2\ G_M(y,\eta)\ c_{m_\delta}(\eta)\right]d\eta \qquad (8)$$

For steady roll, the total rolling moment must be zero. Thus,

$$0 = \int_0^l c(al + \eta)\left[c_{l_{\alpha_e}}(\eta)\ \theta(\eta) + \frac{pb}{2V}\ c_{l_p}(\eta) + \delta\ c_{l_\delta}(\eta)\right]d\eta \qquad (9)$$

## SOLUTION OF AEROELASTIC EQUATIONS

### Galerkin-Type Procedure

If, for a particular configuration, the values of $q$ and $\delta$ are given, equations (8) and (9) can be solved simultaneously to yield the values of twist $\theta$ and rate of roll $pb/2V$ for the elastic aircraft. In reference 1, these equations were solved by a collocation procedure that involved the solution of high-order matrix equations. A method that is considerably simpler (which takes the form of the Galerkin method) is used herein. The solution proceeds as follows:

Let

$$\theta(y) = k\,\theta_1(y) \tag{10}$$

where $\theta_1(y)$ is an approximation of the actual expected twist shape. Note that, although the Galerkin solution generally involves the use of a series of such functions, only one term is used for this particular application.

If the approximation for $\theta(y)$ (eq. (10)) is introduced into equation (8), and the resulting equation is multiplied by $\theta_1(y)$ and integrated over the exposed semispan of the wing, the following equation is obtained:

$$\begin{aligned} k\int_0^l \theta_1^2(y)\,dy = {} & qk\int_0^l \int_0^l \theta_1(y)\left[c\,G_L(y,\eta)\,c_{l_{\alpha_e}}(\eta) + c^2\,G_M(y,\eta)\,c_{m_{\alpha_e}}(\eta)\right]\theta_1(\eta)\,d\eta\,dy\ + \\ & q\,\frac{pb}{2V}\int_0^l \int_0^l \theta_1(y)\left[c\,G_L(y,\eta)\,c_{l_p}(\eta) + c^2\,G_M(y,\eta)\,c_{m_p}(\eta)\right]d\eta\,dy\ + \\ & q\delta\int_0^l \int_0^l \theta_1(y)\left[c\,G_L(y,\eta)\,c_{l_\delta}(\eta) + c^2\,G_M(y,\eta)\,c_{m_\delta}(\eta)\right]d\eta\,dy \end{aligned} \tag{11}$$

Introduction of equation (10) into equation (9) yields

$$0 = k\int_0^l c(al + \eta)\ c_{l_{\alpha_e}}(\eta)\ \theta_1(\eta)\ d\eta + \frac{pb}{2V}\int_0^l c(al + \eta)\ c_{l_p}(\eta)\ d\eta +$$

$$\delta\int_0^l c(al + \eta)\ c_{l_\delta}(\eta)\ d\eta \tag{12}$$

Dividing equations (11) and (12) by $\delta$ yields two simultaneous equations in two unknowns, $k/\delta$ and $\frac{pb}{2V}\Big/\delta$. The quantity that is actually sought is the latter; solving for this quantity gives

$$\frac{pb}{2V}\Big/\delta = -\frac{AC_{l_\delta} - q\left(B_\theta C_{l_\delta} - B_\delta C_{l_\theta}\right)}{AC_{l_p} - q\left(B_\theta C_{l_p} - B_p C_{l_\theta}\right)} \tag{13}$$

where

$$A = \int_0^l \theta_1^{\,2}(y)\ dy \tag{13a}$$

$$\left.\begin{aligned}
B_\theta &= \int_0^l\int_0^l \theta_1(y)\left[c\ G_L(y,\eta)\ c_{l_{\alpha_e}}(\eta) + c^2\ G_M(y,\eta)\ c_{m_{\alpha_e}}(\eta)\right]\theta_1(\eta)\ d\eta\ dy \\
B_p &= \int_0^l\int_0^l \theta_1(y)\left[c\ G_L(y,\eta)\ c_{l_p}(\eta) + c^2\ G_M(y,\eta)\ c_{m_p}(\eta)\right]d\eta\ dy \\
B_\delta &= \int_0^l\int_0^l \theta_1(y)\left[c\ G_L(y,\eta)\ c_{l_\delta}(\eta) + c^2\ G_M(y,\eta)\ c_{m_\delta}(\eta)\right]d\eta\ dy
\end{aligned}\right\} \tag{13b}$$

and

$$\left.\begin{aligned}
C_{l_\theta} &= \frac{1}{2l^2\bar{c}(1 + a)} \int_0^l c(al + \eta)\, c_{l_{\alpha_e}}(\eta)\, \theta_1(\eta)\, d\eta \\
C_{l_p} &= \frac{1}{2l^2\bar{c}(1 + a)} \int_0^l c(al + \eta)\, c_{l_p}(\eta)\, d\eta \\
C_{l_\delta} &= \frac{1}{2l^2\bar{c}(1 + a)} \int_0^l c(al + \eta)\, c_{l_\delta}(\eta)\, d\eta
\end{aligned}\right\} \tag{13c}$$

The quantities $C_{l_\theta}$, $C_{l_p}$, and $C_{l_\delta}$ are actually the rolling-moment coefficients (based on exposed wing area) resulting from a unit mode shape $\theta_1$, a unit $pb/2V$, and a unit $\delta$, respectively.

Rolling effectiveness.- The rolling effectiveness $\phi$ is defined as the ratio of the rate of roll of the flexible airplane to the rate of roll that would occur if the airplane were rigid. The rate of roll for the rigid wing is given by

$$\left(\frac{pb}{2V}\Big/\delta\right)_R = -\frac{C_{l_\delta}}{C_{l_p}}$$

Therefore, the rolling effectiveness is

$$\phi = \frac{\left(\frac{pb}{2V}\Big/\delta\right)_F}{\left(\frac{pb}{2V}\Big/\delta\right)_R} = \frac{1 - q\left(\frac{B_\theta}{A} - \frac{B_\delta}{A}\frac{C_{l_\theta}}{C_{l_\delta}}\right)}{1 - q\left(\frac{B_\theta}{A} - \frac{B_p}{A}\frac{C_{l_\theta}}{C_{l_p}}\right)} \tag{14}$$

Aileron reversal.- The dynamic pressure at aileron reversal can be found by setting $\phi$ equal to zero. The result is

$$q_{rev} = \frac{1}{\frac{B_\theta}{A} - \frac{B_\delta}{A}\frac{C_{l_\theta}}{C_{l_\delta}}} \tag{15}$$

Twist mode shape.- The mode shape $\theta_1$ should be a reasonably good approximation to the actual twist expected. One possible shape that may meet this requirement is the twist that would result from the application of the aileron loads only. This shape is given by

$$\theta_1(y) = \int_0^l \left[c\ G_L(y,\eta)\ c_{l_\delta}(\eta) + c^2\ G_M(y,\eta)\ c_{m_\delta}(\eta)\right] d\eta \tag{16}$$

Simplifications for wings with elastic axis.- When an elastic axis exists, the structural equilibrium is expressed by equation (2) rather than equation (1). Therefore, the quantities $B_\theta$, $B_p$, and $B_\delta$ (in eqs. (13b) and the definition of the twist mode shape (eq. (16)) can be altered by deleting the terms containing $G_L(y,\eta)$ in equations (13b) and (16) and by replacing $c_{m_{\alpha_e}}$, $c_{m_p}$, and $c_{m_\delta}$ with

$$\left.\begin{aligned} c_{q_{\alpha_e}} &= c_{m_{\alpha_e}} - e\ c_{l_{\alpha_e}} \\ c_{q_p} &= c_{m_p} - e\ c_{l_p} \\ c_{q_\delta} &= c_{m_\delta} - e\ c_{l_\delta} \end{aligned}\right\} \tag{17}$$

## Numerical Evaluation of Integrals

The Galerkin procedure just presented involves the calculation of a number of integrals. In this section, these integrals are found numerically by using Simpson's rule with 10 equal spanwise intervals. Matrix notation is used to facilitate the representation of the integrals. The integration scheme and the matrix notation are similar to those set forth in the section entitled "Matrix Operations" in reference 1.

In matrix form, the quantities defined by equations (13a), (13b), and (13c) are

$$A = \left\lfloor \theta_1 \right\rfloor \begin{bmatrix} S \end{bmatrix} \left| \theta_1 \right| \tag{18}$$

$$\left.\begin{aligned}
B_\theta = {} & \left\lfloor \theta_1 \right\rfloor \begin{bmatrix} & & \\ & S & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & G_L & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & S & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & \theta_1 & \\ & & \end{bmatrix} \left| c\ c_{l_{\alpha_e}} \right| + \\
& \left\lfloor \theta_1 \right\rfloor \begin{bmatrix} & & \\ & S & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & G_M & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & S & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & \theta_1 & \\ & & \end{bmatrix} \left| c^2\ c_{m_{\alpha_e}} \right| \\
B_p = {} & \left\lfloor \theta_1 \right\rfloor \begin{bmatrix} & & \\ & S & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & G_L & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & S & \\ & & \end{bmatrix} \left| c\ c_{l_p} \right| + \\
& \left\lfloor \theta_1 \right\rfloor \begin{bmatrix} & & \\ & S & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & G_M & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & S & \\ & & \end{bmatrix} \left| c^2\ c_{m_p} \right| \\
B_\delta = {} & \left\lfloor \theta_1 \right\rfloor \begin{bmatrix} & & \\ & S & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & G_L & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & S & \\ & & \end{bmatrix} \left| c\ c_{l_\delta} \right| + \\
& \left\lfloor \theta_1 \right\rfloor \begin{bmatrix} & & \\ & S & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & G_M & \\ & & \end{bmatrix} \begin{bmatrix} & & \\ & S & \\ & & \end{bmatrix} \left| c^2\ c_{m_\delta} \right|
\end{aligned}\right\} \tag{19}$$

and

$$
\left.
\begin{aligned}
C_{l_\theta} &= \frac{1}{2l^2\bar{c}(1 + a)} \left\lfloor al + y \right\rfloor \left\lceil S \right\rfloor \left\lceil \theta_1 \right\rfloor \left| c\, c_{l_{\alpha_e}} \right| \\[2ex]
C_{l_p} &= \frac{1}{2l^2\bar{c}(1 + a)} \left\lfloor al + y \right\rfloor \left\lceil S \right\rfloor \left| c\, c_{l_p} \right| \\[2ex]
C_{l_\delta} &= \frac{1}{2l^2\bar{c}(1 + a)} \left\lfloor al + y \right\rfloor \left\lceil S \right\rfloor \left| c\, c_{l_\delta} \right|
\end{aligned}
\right\} \tag{20}
$$

In these definitions, $\left\lfloor \theta_1 \right\rfloor$, $\left\lceil \theta_1 \right\rfloor$, and $\left| \theta_1 \right|$ are row, diagonal, and column matrices, respectively, made up of the assumed twist shape; $\left\lceil S \right\rfloor$ is an integrating matrix given by

$$
\left\lceil S \right\rfloor = \frac{l}{30}
\begin{bmatrix}
1 & & & & & & & & & & \\
& 4 & & & & & & & & & \\
& & 2 & & & & & & & & \\
& & & 4 & & & & & & & \\
& & & & 2 & & & & & & \\
& & & & & 4 & & & & & \\
& & & & & & 2 & & & & \\
& & & & & & & 4 & & & \\
& & & & & & & & 2 & & \\
& & & & & & & & & 4 & \\
& & & & & & & & & & 1
\end{bmatrix} \tag{21}
$$

This particular integrating matrix, of course, has been obtained by applying Simpson's rule. Other schemes for numerical integration could be used by appropriately modifying $\left\lceil S \right\rfloor$. It is questionable, however, whether any increase in accuracy obtained by changing the integration rule would be worthwhile for this problem. Note that, for simplicity, the subscript notation used in reference 1 to denote the location of elements in the matrices has been dropped in this paper.

The mode shape $\theta_1$ (eq. (16)) is written in matrix form as

$$\left|\theta_1\right| = \left[G_L\right]\left[\diagdown S \diagdown\right]\left|c\, c_{l_\delta}\right| + \left[G_M\right]\left[\diagdown S \diagdown\right]\left|c^2\, c_{m_\delta}\right| \quad (22)$$

When an elastic axis exists, the matrix formulation is considerably simplified by the same procedure mentioned previously; that is, in equations (19) and (22), the terms involving $\left[G_L\right]$ should be deleted, and the column matrices involving coefficients of moments about the y-axis should be replaced by coefficients of moment about the elastic axis.

## APPLICATION

### Computational Procedure

As can be seen from the preceding analysis, the requisite quantities for determining the aeroelastic effects on the roll of a particular aircraft are the structural influence coefficients $\left[G_L\right]$ and $\left[G_M\right]$ and the aerodynamic derivatives $c_{l_p}$, $c_{m_p}$, $c_{l_\delta}$, and $c_{m_\delta}$. (The derivatives $c_{l_{\alpha_e}}$ and $c_{m_{\alpha_e}}$ are given in terms of $c_{l_p}$ and $c_{m_p}$, respectively, by eqs. (7).) The influence coefficients are dependent on the structure only, but the aerodynamic derivatives vary with Mach number. If a range of Mach numbers is to be covered, therefore, these derivatives must be calculated anew for each value of $M_o$.

Not only the aerodynamic derivatives but also the mode shape $\theta_1$ (as calculated from eq. (22)) varies with $M_o$. The variation of the derivatives is unavoidable; the variation of $\theta_1$ can often be circumvented, however, by calculating $\theta_1$ for a particular Mach number and then by using this same mode shape for the other values of $M_o$. This process would involve little loss in accuracy provided that the lift and pitching-moment distributions due to aileron deflection do not change radically with Mach number.

With $\theta_1$ determined, the calculation of the values of the quantities appearing in equations (13), (14), and (15) proceeds in a straight forward fashion. (The matrix multiplications in eqs. (18), (19), and (20) should be performed from left to right because the variable aerodynamic derivatives are the last terms.) With these quantities determined, the dynamic pressure at aileron reversal $q_{rev}$ can be calculated from equation (15), and the rolling effectiveness $\phi$ for other values of $q$ can then be computed from equation (14). This process is repeated for each Mach number until the entire range is covered.

## Sample Calculations

The method derived in this report is applied to two example configurations. Both of these aircraft have two flexible rectangular wings mounted diametrically on a long cylindrical fuselage; both aircraft have full-span, 0.2-chord, trailing-edge ailerons. Attention is restricted to the supersonic-speed regime.

The two configurations are shown in figure 2 and are designated as models 1 and 2; the wings of both models have the same plan-form aspect ratio of $l/c = 1.5$. The wings differ, however, in that model 1 has a rectangular cross section with a thickness ratio of 0.02, whereas model 2 has an NACA 65A003 cross-sectional shape. The models also differ in that the value of $a$, the ratio of fuselage radius to exposed wing semispan, is 0.2 for model 1 and 0.236 for model 2. Both wings were assumed to be made of solid aluminum alloy.

The torsional influence coefficients for the two models are given in table I. These influence coefficients were obtained from an approximate plate theory which is essentially the same as that of reference 4; however, the analyses of the two models were slightly different: For model 1, the root was assumed to be completely clamped, and no account was made for the stiffening flange effect of the bent-up aileron. For model 2, some root flexibility was allowed, and the flange effect of the aileron was taken into account approximately. In both cases, the analyses indicated the existence of an elastic axis; therefore, only the torsional influence coefficients due to torque $\left[G_M\right]$ are given in table I.

As is evident from the preceding discussion, model 1 represents an idealized configuration (actually the same as the model considered in ref. 1), and model 2 represents a more realistic aircraft. The following table summarizes some of the information necessary for analyzing the models:

| | Model 1 | Model 2 |
|---|---|---|
| $l/c$ . . . . . . . . . . . . . . . . . . . . . . . . . . . . | 1.5 | 1.5 |
| $c_a/c$ . . . . . . . . . . . . . . . . . . . . . . . . . . . | 0.2 | 0.2 |
| a . . . . . . . . . . . . . . . . . . . . . . . . . . . . . | 0.2 | 0.236 |
| e . . . . . . . . . . . . . . . . . . . . . . . . . . . . . | 0 | 0.0485 |

No absolute dimensions have been specified because, as is shown in reference 1, only the ratios are needed to analyze the aeroelastic-rolling problem.

The aerodynamic rolling derivatives were obtained for $M_o = 1.108$, 1.202, 1.338, 1.667, and 2.848 for which tables are available in reference 1. For illustrative purposes, the values of $c_{l_p}$ and $c_{q_p}$ for the two models at $M_o = 1.202$ are given in table II for values of $y/l$ between 0 and 1.0 in 0.1 increments. The corresponding values of $c_{l_{\alpha_e}}$ and $c_{q_{\alpha_e}}$ are also included in this table.

The aileron derivatives $c_{l_\delta}$ and $c_{q_\delta}$ were found by assuming that two-dimensional theory is adequate for all stations except at the tip where the loads are zero. With this assumption, the aileron loads become

$$\left.\begin{aligned} c_{l_\delta} &= \frac{4}{\beta}\frac{c_a}{c} \qquad \frac{y}{l} = 0, 0.1, 0.2, \ldots 0.9 \\ c_{l_\delta} &= 0 \qquad \frac{y}{l} = 1.0 \end{aligned}\right\} \tag{23}$$

$$\left.\begin{aligned} c_{q_\delta} &= -\frac{2}{\beta}\frac{c_a}{c}\left(1 + 2e - \frac{c_a}{c}\right) \qquad \frac{y}{l} = 0, 0.1, 0.2, \ldots 0.9 \\ c_{q_\delta} &= 0 \qquad \frac{y}{l} = 1.0 \end{aligned}\right\} \tag{24}$$

where

$$\beta = \sqrt{M_o^{\,2} - 1}$$

The assumed twist mode shape was calculated from equation (22) and is given in table III for the two models. The shapes have been normalized by dividing by the tip ordinate and apply for all Mach numbers as a consequence of the invariance in the shape of the assumed aileron derivatives given by equation (24). In the calculation of the mode shape from equation (22), the simplifications resulting from the existence of an elastic axis were employed. These simplifications were also used wherever else applicable.

With these mode shapes, then, the values of $A$, $B_\theta$, $B_p$, $B_\delta$, $C_{l_\theta}$, $C_{l_p}$, and $C_{l_\delta}$ were computed from equations (18), (19), and (20) for each Mach number. These quantities are tabulated in table IV for the two models. From these quantities, the dynamic pressure at reversal and the rolling effectiveness $\phi$ can be calculated by equations (15) and (14), respectively.

## RESULTS AND COMPARISONS

The results of the aileron-reversal calculations for the two models are shown by the test-point symbols in figure 3. In this figure the results are given in the form of a plot of the pressure ratio at reversal $\left(\frac{P_h}{P_0}\right)_{rev}$ against Mach number, where $P_h$ is the static pressure at reversal,

$$P_h = \frac{2}{\gamma M_o^2} q_{rev}$$

and $P_0$ is the standard sea-level static pressure, 2,116 lb/sq ft. For comparison, the results obtained by the method of reference 1 are also shown in the figure. The agreement is seen to be very good, particularly at the higher Mach numbers.

For the two example configurations considered, the aileron-reversal results alone provide an adequate test of the accuracy of the method of this paper. This fact arises from the virtual linearity of the variation of rolling effectiveness $\phi$ with dynamic pressure (or pressure ratio). The rolling-effectiveness curves for model 1 are given in reference 1 and are almost linear. For model 2, the calculations by the method of reference 1 exhibited even better linearity. Similar degrees of linearity also result from the method contained herein. For the above reasons, no rolling-effectiveness plots are included in this paper.

## CONCLUDING REMARKS

The simplified method outlined in this paper for the prediction of aeroelastic effects on roll is evidently capable of yielding results that compare favorably with those of highly refined methods. Although the method has been tested for only two configurations at supersonic speeds, there is no reason to suspect that the agreement for other configurations at other speeds would be significantly worse.

The foregoing discussion is not meant to imply that this method is applicable in all cases. For instance, one of the most worrisome problems facing the aeroelastician is that of chordwise distortion of the wing; the effects of chordwise distortion, which often appears in wings with very low aspect ratio, are not considered in this paper. In addition, the single-mode Galerkin-type approach used herein may not be good enough for some configurations; with a highly swept wing having inboard ailerons, for example, the actual twist distribution changes radically with dynamic pressure, and no single assumed twist mode shape could be expected to yield good results over the entire range of dynamic pressure. However, such configurations are rarely encountered. Lastly, it is clear that any results obtained by this method would be only as good as the structural and aerodynamic ingredients introduced into the calculations. For this reason, the methods of structural and aerodynamic analysis must be reliable. In some cases - at transonic speeds, for instance - resort would have to be made to experiment to determine parts of the basic information.

Although the attention throughout this report has been confined to the rolling problem, the same type of approach could be used for other static-aeroelastic problems such as torsional divergence and center-of-pressure shift.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., December 16, 1954.

REFERENCES

1. Hedgepeth, John M., and Kell, Robert J.: Rolling Effectiveness and Aileron Reversal of Rectangular Wings at Supersonic Speeds. NACA TN 3067, 1954.

2. Schuerch, Hans U.: Structural Analysis of Swept, Low Aspect Ratio, Multispar Aircraft Wings. Aero. Eng. Rev., vol. 11, no. 11, Nov. 1952, pp. 34-41.

3. Levy, Samuel: Computation of Influence Coefficients for Aircraft Structures With Discontinuities and Sweepback. Jour. Aero. Sci., vol. 14, no. 10, Oct. 1947, pp. 547-560.

4. Reissner, Eric, and Stein, Manuel: Torsion and Transverse Bending of Cantilever Plates. NACA TN 2369, 1951.

5. Radok, J. R. M.: Aileron Reversal and Divergence of Swept Wings With Special Consideration of the Relevant Aerodynamic and Elastic Characteristics. Rep. No. 55, College of Aero., Cranfield (British), Mar. 1952.

6. Martin, John C., and Jeffreys, Isabella: Span Load Distributions Resulting From Angle of Attack, Rolling, and Pitching for Tapered Sweptback Wings With Streamwise Tips - Supersonic Leading and Trailing Edges. NACA TN 2643, 1952.

7. Hannah, Margery E., and Margolis, Kenneth: Span Load Distributions Resulting From Constant Angle of Attack, Steady Rolling Velocity, Steady Pitching Velocity, and Constant Vertical Acceleration for Tapered Sweptback Wings With Streamwise Tips - Subsonic Leading Edges and Supersonic Trailing Edges. NACA TN 2831, 1952.

8. Lagerstrom, P. A., and Graham, Martha E.: Linearized Theory of Supersonic Control Surfaces. Jour. Aero. Sci., vol. 16, no. 1, Jan. 1949, pp. 31-34.

9. Diederich, Franklin W.: A Simple Approximate Method for Calculating Spanwise Lift Distributions and Aerodynamic Influence Coefficients at Subsonic Speeds. NACA TN 2751, 1952.

TABLE I

TORSIONAL INFLUENCE COEFFICIENTS FOR

EXAMPLE CONFIGURATIONS

(a) Model 1; $\frac{GJ}{c^4} = 5,151$ lb/sq ft

$$\left[a_M\right] = \frac{1}{GJ}\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.007838 & 0.015492 & 0.019693 & 0.021999 & 0.023266 & 0.023964 & 0.024351 & 0.024570 & 0.024704 & 0.024802 \\
0 & 0.015492 & 0.042839 & 0.061202 & 0.071283 & 0.076821 & 0.079869 & 0.081560 & 0.082520 & 0.083104 & 0.083531 \\
0 & 0.019693 & 0.061202 & 0.102832 & 0.129038 & 0.143435 & 0.151360 & 0.155756 & 0.158250 & 0.159770 & 0.160879 \\
0 & 0.021999 & 0.071283 & 0.129038 & 0.179596 & 0.210725 & 0.227862 & 0.237366 & 0.242760 & 0.246047 & 0.248444 \\
0 & 0.023266 & 0.076821 & 0.143435 & 0.210725 & 0.266557 & 0.300659 & 0.319569 & 0.330304 & 0.336843 & 0.341613 \\
0 & 0.023964 & 0.079869 & 0.151360 & 0.227862 & 0.300659 & 0.359659 & 0.395765 & 0.416260 & 0.428745 & 0.437852 \\
0 & 0.024351 & 0.081560 & 0.155756 & 0.237366 & 0.319569 & 0.395765 & 0.457123 & 0.495419 & 0.518749 & 0.535766 \\
0 & 0.024570 & 0.082520 & 0.158250 & 0.242760 & 0.330304 & 0.416260 & 0.495419 & 0.560052 & 0.603147 & 0.634583 \\
0 & 0.024704 & 0.083104 & 0.159770 & 0.246047 & 0.336843 & 0.428745 & 0.518749 & 0.603147 & 0.676155 & 0.733861 \\
0 & 0.024802 & 0.083531 & 0.160879 & 0.248444 & 0.341613 & 0.437852 & 0.535766 & 0.634583 & 0.733861 & 0.833335
\end{bmatrix}$$

(b) Model 2; $\frac{GJ}{c^4} = 2,413$ lb/sq ft

$$\left[a_M\right] = \frac{1}{GJ}\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.045956 & 0.071161 & 0.082136 & 0.086915 & 0.088996 & 0.089903 & 0.090298 & 0.090473 & 0.090554 & 0.090601 \\
0 & 0.071161 & 0.131088 & 0.162377 & 0.176000 & 0.181933 & 0.184518 & 0.185646 & 0.186144 & 0.186375 & 0.186508 \\
0 & 0.082136 & 0.162377 & 0.226257 & 0.259266 & 0.273642 & 0.279904 & 0.282638 & 0.283843 & 0.284403 & 0.284726 \\
0 & 0.086915 & 0.176000 & 0.259266 & 0.324466 & 0.358053 & 0.372685 & 0.379071 & 0.381888 & 0.383195 & 0.383950 \\
0 & 0.088996 & 0.181933 & 0.273642 & 0.358053 & 0.423756 & 0.457575 & 0.472337 & 0.478846 & 0.481867 & 0.483612 \\
0 & 0.089903 & 0.184518 & 0.279904 & 0.372685 & 0.457575 & 0.523516 & 0.557505 & 0.572492 & 0.579448 & 0.583464 \\
0 & 0.090298 & 0.185646 & 0.282638 & 0.379071 & 0.472337 & 0.557505 & 0.623719 & 0.658175 & 0.674166 & 0.683400 \\
0 & 0.090473 & 0.186144 & 0.283843 & 0.381888 & 0.478846 & 0.572492 & 0.658175 & 0.725414 & 0.762157 & 0.783372 \\
0 & 0.090554 & 0.186375 & 0.284403 & 0.383195 & 0.481867 & 0.579448 & 0.674166 & 0.762157 & 0.834629 & 0.883359 \\
0 & 0.090601 & 0.186508 & 0.284726 & 0.383950 & 0.483612 & 0.583464 & 0.683400 & 0.783372 & 0.883359 & 0.983351
\end{bmatrix}$$

TABLE II

AERODYNAMIC COEFFICIENTS FOR $M_o = 1.202$

| $y/l$ | $c_{l_p}(y/l)$ | $c_{q_p}(y/l)$ | $c_{l_{\alpha_e}}(y/l)$ | $c_{q_{\alpha_e}}(y/l)$ |
|---|---|---|---|---|
| Model 1 | | | | |
| 0 | -1.000000 | 0 | 6.000000 | 0 |
| .1 | -1.415569 | -.038833 | 5.662275 | .155333 |
| .2 | -1.749190 | -.105254 | 5.247568 | .315763 |
| .3 | -2.015077 | -.183793 | 4.836184 | .441105 |
| .4 | -2.212072 | -.266137 | 4.424151 | .532274 |
| .5 | -2.333569 | -.344835 | 4.000430 | .591149 |
| .6 | -2.367374 | -.411680 | 3.551063 | .617521 |
| .7 | -2.292641 | -.456080 | 3.056844 | .608105 |
| .8 | -2.070005 | -.461762 | 2.484005 | .554115 |
| .9 | -1.604045 | -.395341 | 1.749870 | .431281 |
| 1.0 | 0 | 0 | 0 | 0 |
| Model 2 | | | | |
| 0 | -1.145629 | 0.055563 | 6.000000 | -0.291000 |
| .1 | -1.546673 | .036198 | 5.689547 | -.133159 |
| .2 | -1.865921 | -.014696 | 5.289627 | .041662 |
| .3 | -2.117637 | -.080833 | 4.883205 | .186398 |
| .4 | -2.300731 | -.154066 | 4.471233 | .299412 |
| .5 | -2.408604 | -.227246 | 4.044883 | .381624 |
| .6 | -2.429104 | -.292778 | 3.591357 | .432862 |
| .7 | -2.341335 | -.341137 | 3.091764 | .450477 |
| .8 | -2.105803 | -.358047 | 2.512328 | .427169 |
| .9 | -1.626493 | -.314764 | 1.769672 | .342472 |
| 1.0 | 0 | 0 | 0 | 0 |

TABLE III

TWIST MODE SHAPES USED IN GALERKIN-TYPE SOLUTION

| $y/l$ | $\theta_1$ for - | |
|---|---|---|
| | Model 1 | Model 2 |
| 0 | 0 | 0 |
| .1 | .057166 | .167091 |
| .2 | .182638 | .331836 |
| .3 | .332681 | .483678 |
| .4 | .482540 | .616914 |
| .5 | .619788 | .730274 |
| .6 | .737187 | .822233 |
| .7 | .832533 | .893651 |
| .8 | .905312 | .944649 |
| .9 | .959032 | .978453 |
| 1.0 | 1.000000 | 1.000000 |

TABLE IV

VALUES OF $A$, $B_\theta$, $B_p$, $B_\delta$, $C_{l_\theta}$, $C_{l_p}$, AND $C_{l_\delta}$

| $M_o$ | $A \times \frac{1}{l}$ | $B_\theta \times \frac{GJ}{l^3c^2}$ | $B_p \times \frac{GJ}{l^3c^2}$ | $B_\delta \times \frac{GJ}{l^3c^2}$ | $C_{l_\theta}$ | $C_{l_p}$ | $C_{l_\delta}$ |
|---|---|---|---|---|---|---|---|
| Model 1 | | | | | | | |
| 1.108 | 0.423398 | 0.087669 | -0.084086 | -0.094806 | 0.522170 | -0.526021 | 0.462000 |
| 1.202 | .423398 | .056242 | -.053038 | -.067719 | .543180 | -.552164 | .330000 |
| 1.338 | .423398 | .035716 | -.033229 | -.050789 | .510437 | -.514456 | .247500 |
| 1.667 | .423398 | .016822 | -.015677 | -.033859 | .420363 | -.417449 | .165000 |
| 2.848 | .423398 | .004079 | -.003882 | -.016930 | .252679 | -.248333 | .082500 |
| Model 2 | | | | | | | |
| 1.108 | 0.510760 | 0.128009 | -0.112045 | -0.174174 | 0.630158 | -0.562615 | 0.472194 |
| 1.202 | .510760 | .067671 | -.058776 | -.124410 | .649197 | -.581382 | .337281 |
| 1.338 | .510760 | .032038 | -.028037 | -.093307 | .603209 | -.537432 | .252961 |
| 1.667 | .510760 | .005056 | -.005035 | -.062205 | .488809 | -.433484 | .168641 |
| 2.848 | .510760 | -.006153 | .005059 | -.031102 | .288649 | -.256513 | .084320 |

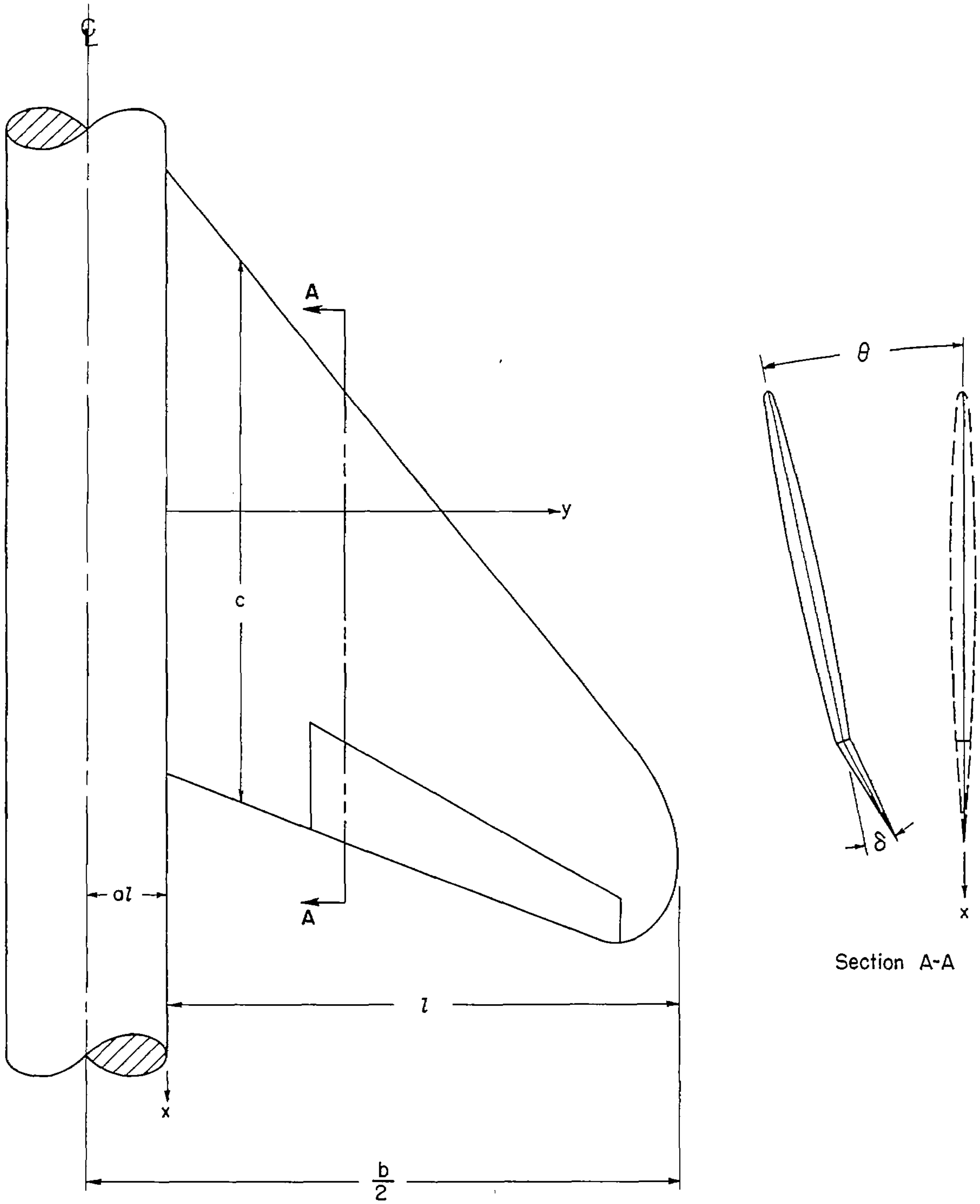


Figure 1.- Configuration considered in aeroelastic analysis.

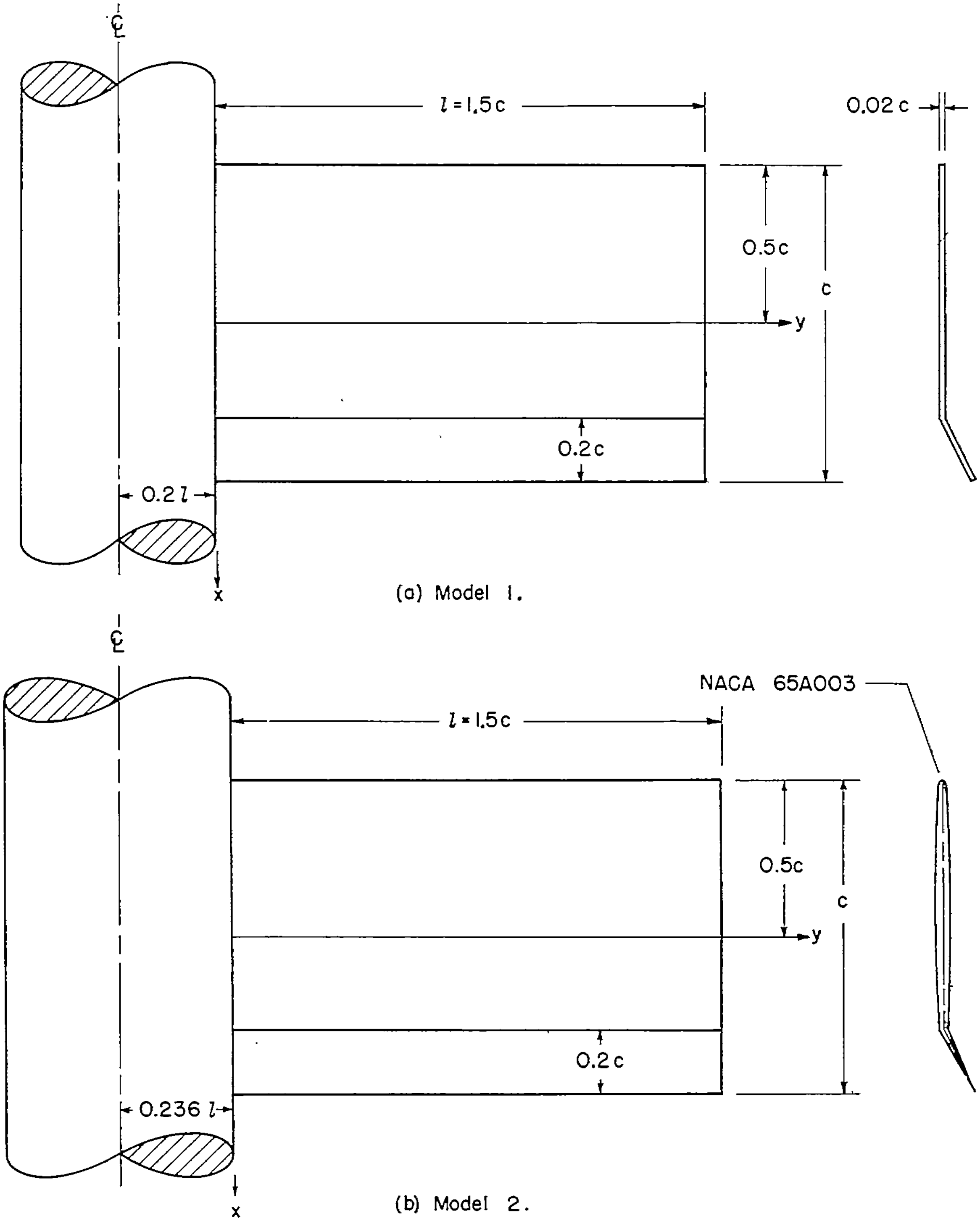


Figure 2.- Example configurations.

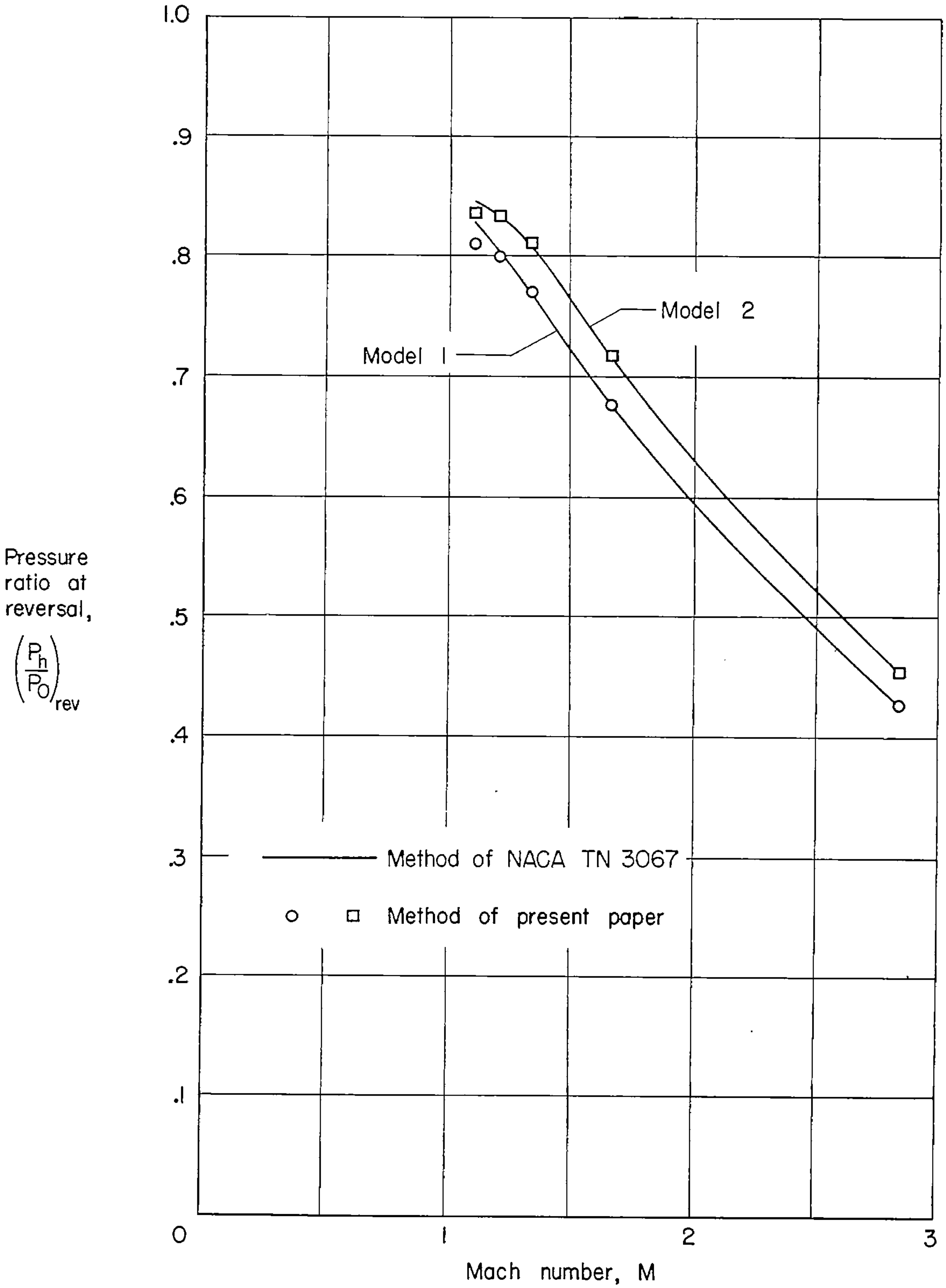


Figure 3.- Variation of pressure ratio at reversal with Mach number.